OpenStax: Integrated Rate Laws

Example 12.6
The rate constant for the first-order decomposition of cyclobutane, C_4H_8 at 500 °C is 9.2×10^{-3} s^{-1}:

$C_4H_8 \longrightarrow 2C_2H_4$

How long will it take for 80.0% of a sample of C_4H_8 to decompose?

Use the integrated form of the first-order rate law $\ln([A]_0/[A]) = kt$ to answer questions regarding time:

1.7×10^2 s

Iodine-131 is a radioactive isotope that is used to diagnose and treat some forms of thyroid cancer.

Iodine-131 decays to xenon-131 according to the equation:

I-131 ⟶ Xe-131 + electron

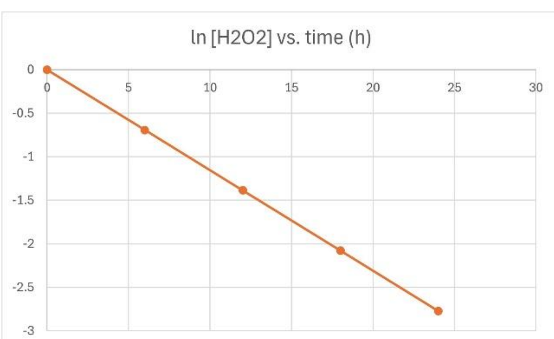
The decay is first-order with a rate constant of 0.138 d^{-1}. All radioactive decay is first order. How many days will it take for 90% of the iodine−131 in a 0.500 M solution of this substance to decay to Xe-131?

16.7 days

Example 12.7

Show that the relationship between the $\ln[H_2O_2]$ and time is linear by graphing.

Trial	Time, h	$[H_2O_2]$ (M)	$\ln[H_2O_2]$
1	0	1.000	0.0
2	6.00	0.500	-0.693
3	12.00	0.250	-1.386
4	18.00	0.125	-2.079
5	24.00	0.0625	-2.772



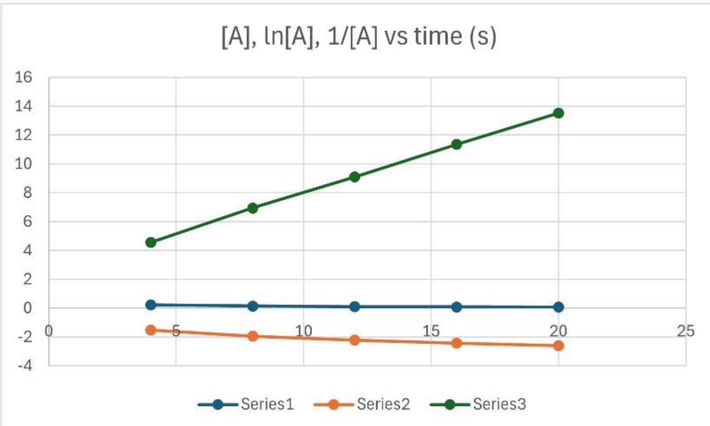
Determine the rate constant from this data.

$k = -\text{slope} = 1.155 \times 10^{-1}\ h^{-1}$

Example 12.8

a. Graph the following data to determine whether the reaction A ⟶ B + C is first order.

Trial	Time (s)	[A]	ln[A]	1/[A]
1	4.0	0.220	-1.514	4.545
2	8.0	0.144	-1.938	6.944
3	12.0	0.110	-2.207	9.091
4	16.0	0.088	-2.430	11.36
5	20.0	0.074	-2.603	13.51

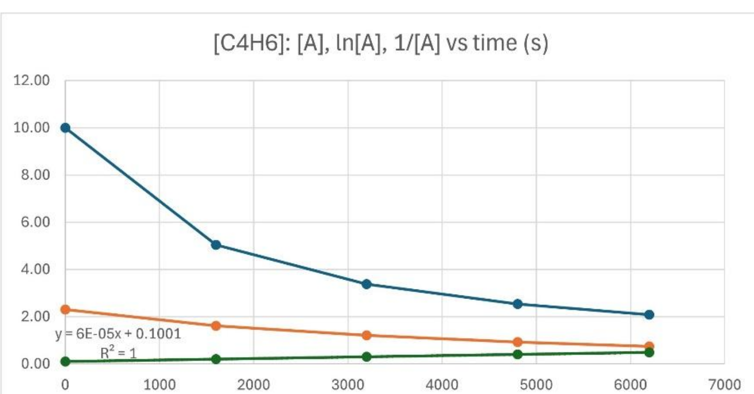


The data represents a second order rate. k = 0.600 L^2 mol^{-2} s^{-1}

b. Test these data to confirm that this dimerization reaction is second-order.

2 $C_4H_6(g)$ -> $C_8H_{12}(g)$

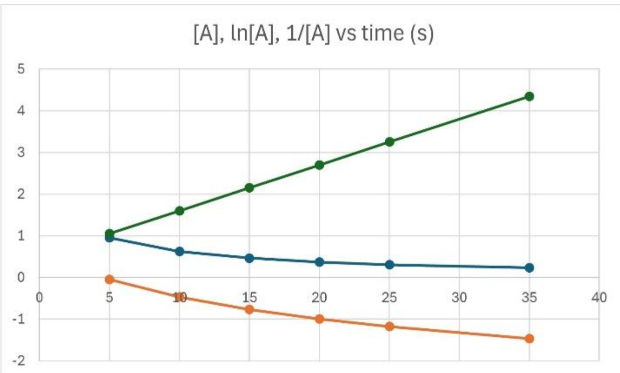
Trial	Time (s)	$[C_4H_6]$ (M)	$\ln[C_4H_6]$	$1/[C_4H_6]$
1	0	1.00×10^{-2}	-4.61	100
2	1600	5.04×10^{-3}	-5.29	198
3	3200	3.37×10^{-3}	-5.69	297
4	4800	2.53×10^{-3}	-5.98	395
5	6200	2.08×10^{-3}	-6.18	481



The data represents a second order rate. k = 0.06125 L^2 mol^{-2} s^{-1}

c. Does the following data fit a second-order rate law?

Trial	Time (s)	[A] (M)	ln[A]	1/[A]
1	5	0.952	-0.0492	1.05
2	10	0.625	-0.470	1.60
3	15	0.465	-0.766	2.15
4	20	0.370	-0.994	2.70
5	25	0.308	-1.18	3.25
6	35	0.230	-1.47	4.35



The data represents a second order rate. k = 0.110

4. A study of the rate of dimerization of C_4H_6 given the data shown in the table:

$2C_4H_6 \longrightarrow C_8H_{12}$

Time (s)	0	1600	3200	4800	6200
$[C_4H_6]$ (M)	1.00×10^{-2}	5.04×10^{-3}	3.37×10^{-3}	2.53×10^{-3}	2.08×10^{-3}

(a) Determine the average rate of dimerization between 0 s and 1600 s, and between 1600 s and 3200 s.

$Rate_{12} = - (5.04 \times 10^{-3} – 1.00 \times 10^{-2}) / (1600 - 0) = 4.96 \times 10^{-3} / 1600$
$Rate_{12} = 3.10 \times 10^{-6}\ L^2\ mol^{-2}\ s^{-1}$

$Rate_{23} = - (3.37 \times 10^{-3} – 5.04 \times 10^{-2}) / (3200 - 1600) = 1.67 \times 10^{-3} / 1600$
$Rate_{23} = 1.04 \times 10^{-6}\ L^2\ mol^{-2}\ s^{-1}$

(b) Estimate the instantaneous rate of dimerization at 3200 s from a graph of time versus $[C_4H_6]$. What are the units of this rate?

$Rate_3 = - (2.53 \times 10^{-3} - 5.04 \times 10^{-2}) / (4800-1600) = 2.51 \times 10^{-3} / 3200$
$Rate_3 = 7.84 \times 10^{-7}\ mol\ L^{-1}\ s^{-1}$

(c) Determine the average rate of formation of C_8H_{12} at 1600 s and the instantaneous rate of formation at 3200 s from the rates found in parts (a) and (b).

$Rate_2 = - 0.5 * (3.37 \times 10^{-3} – 1.00 \times 10^{-3}) / (3200 – 0) = 0.5 * 6.63 \times 10^{-3} / 3200$
$Rate_2 = 1.04 \times 10^{-6}\ mol\ L^{-1}\ s^{-1}$

$Rate_3 = - 0.5 * (2.53 \times 10^{-3} – 5.04 \times 10^{-3}) / (4800 – 1600) = 0.5 * 2.51 \times 10^{-3} / 3200$
$Rate_3 = 3.92 \times 10^{-7}$ mol L^{-1} s^{-1}

5. A study of the rate of the reaction represented as 2A ⟶ B gave the following data:

Time (s)	0.0	5.0	10.0	15.0	20.0	25.0	35.0
[A] (M)	1.00	0.775	0.625	0.465	0.360	0.285	0.230

(a) Determine the average rate of disappearance of A between 0.0 s and 10.0 s, and between 10.0 s and 20.0 s.

$Rate_2 = - (0.625 – 1.00) / (10.0 – 0.0) = 0.375 / 10.0$
$Rate_2 = 3.75 \times 10^{-2}$ mol L^{-1} s^{-1}

(b) Estimate the instantaneous rate of disappearance of A at 15.0 s from a graph of time versus [A]. What are the units of this rate?

$Rate_4 = - (0.0.360 – 0.625) / (20.0 – 10.0) = 0.265 / 10.0$
$Rate_4 = 2.65 \times 10^{-2}$ mol L^{-1} s^{-1}

(c) Use the rates found in parts (a) and (b) to determine the average rate of formation of B between 0.00 s and 10.0 s, and the instantaneous rate of formation of B at 15.0 s.

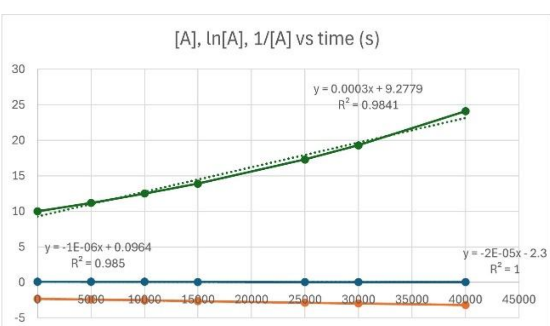
$Rate_3 = 0.5 * 3.75 \times 10^{-2}$ mol L^{-1} s^{-1}
$Rate_3 = 1.875 \times 10^{-2}$ mol L^{-1} s^{-1}

$k_4 = 0.5 * 2.65 \times 10^{-2}$ mol L^{-1} s^{-1}
$k_4 = 1.325 \times 10^{-2}$ mol L^{-1} s^{-1}

33. Use the data provided to graphically determine the order and rate constant of the following reaction:

$SO_2Cl_2 \longrightarrow SO_2 + Cl_2$

Time (s)	0	5.00×10^3	1.00×10^4	1.50×10^4	2.50×10^4	3.00×10^4	4.00×10^4
$[SO_2Cl_2]$ (M)	0.100	0.0896	0.0802	0.0719	0.0577	0.0517	0.0415
$\ln[SO_2Cl_2]$	-2.30	-2.41	-2.52	-2.63	-2.85	-2.96	-3.18
$1/[SO_2Cl_2]$	10.0	11.2	12.5	13.9	17.3	19.3	24.1



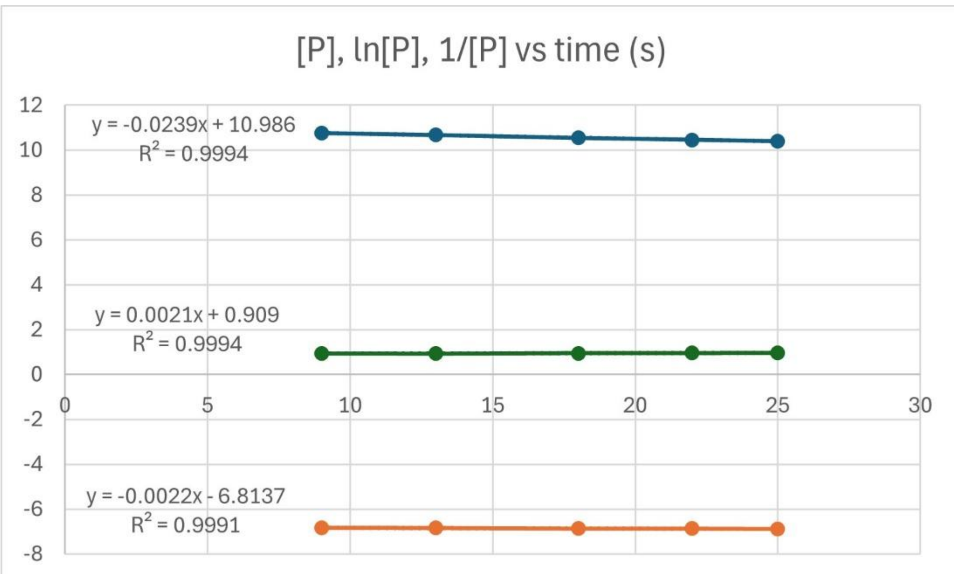
Order [SO_2Cl_2] = 1

k = 2.20 x 10^{-5} s^{-1}

34. Use the data provided in a graphical method to determine the order and rate constant of the following reaction:

$2P \longrightarrow Q + W$

Time (s)	9.0	13.0	18.0	22.0	25.0
[P] (M)	1.077×10^{-3}	1.068×10^{-3}	1.055×10^{-3}	1.046×10^{-3}	1.039×10^{-3}
ln[P]	-6.834	-6.842	-6.854	-6.863	-6.869
1/]P]	928.5	936.3	947.9	956	962.5



Order P = 0, [P] at 0.0 s = 1.0986 x 10^{-3} M

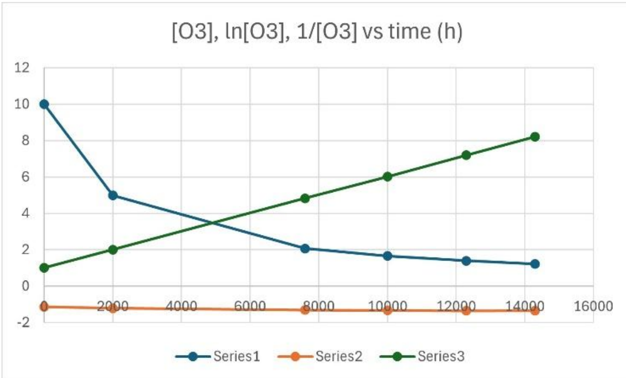
k = 2.39 x 10^{-6} mol L^{-1} s^{-1}

35. Pure ozone decomposes slowly to oxygen

$2O_3(g) \longrightarrow 3O_2(g)$.

Use the data provided in a graphical method and determine the order and rate constant of the reaction.

Time (h)	0	2.0×10^3	7.6×10^3	1.00×10^4	1.23×10^4	1.43×10^4	1.70×10^4
[O_3] (M)	1.00×10^{-5}	4.98×10^{-6}	2.07×10^{-6}	1.66×10^{-6}	1.39×10^{-6}	1.22×10^{-6}	1.22×10^{-6}
ln[O_3]	-1.15	-1.22	-1.31	-1.33	-1.35	-1.36	-1.36
1/[O_3]	1.00 x 10^5	2.01 x 10^5	4.83 x 10^5	6.02 x 10^5	7.19 x 10^5	8.20 x 10^5	8.20 x 10^5



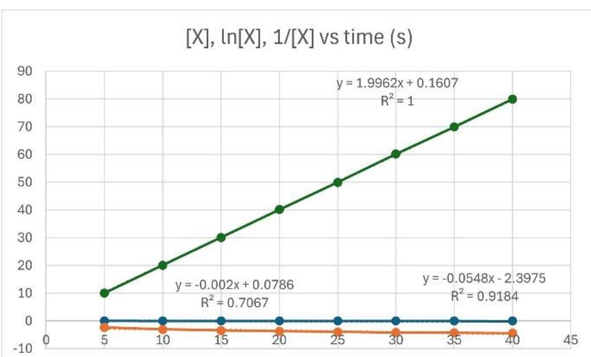
Order O_3 = 2

k = 42.4 L^2 mol^{-2} h^{-1}

36. From the given data, use a graphical method to determine the order and rate constant of the following reaction:

$2X \longrightarrow Y + Z$

Time (s)	5.0	10.0	15.0	20.0	25.0	30.0	35.0	40.0
[X] (M)	0.0990	0.0497	0.0332	0.0249	0.0200	0.0166	0.0143	0.0125
ln[X]	-2.31	-3.00	-3.41	-3.69	-3.91	-4.10	-4.25	-4.38
1/[X]	10.1	20.1	30.1	40.2	50.0	60.2	69.9	80.0



Order X = 2

k = 2.00 L^2 mol^{-2} h^{-1}

49. Nitroglycerine is an extremely sensitive explosive. In a series of carefully controlled experiments, samples of the explosive were heated to 160 °C and their first-order decomposition studied. Determine the average rate constants for each experiment using the following data:

Initial [$C_3H_5N_3O_9$] (M)	4.88	3.52	2.29	1.81	5.33	4.05	2.95	1.72
t (s)	300	300	300	300	180	180	180	180
% Decomposed	52.0	52.9	53.2	53.9	34.6	35.9	36.0	35.4
k	7.80 x 10^{-3}	5.53 x 10^{-3}	3.57 x 10^{-3}	2.78 x 10^{-3}	1.94 x 10^{-2}	1.44 x 10^{-2}	1.05 x 10^{-2}	6.17 x 10^{-3}

Order $C_3H_5N_3O_9$ = 1 (half-life about the same regardless of starting concentration)

$k = 2.5 \times 10^{-3}\ s^{-1}$

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